A STUDY INTO THE DAMAGE
TO RECTANGULAR PLATES SUBJECTED TO
DYNAMIC LOADS

by

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Whatever credit is my own, it pertains to my wife, Leyla.

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Tevfik O. Uran

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ABSTRACT

The objective of this experimental investigation is an attempt to examine technical problems which exist due to our lack of knowledge about the accuracy of plasticity theory when applied to dynamic problems in Naval Engineering. Plastic deformation is a composite function of the physical properties of a specimen, strain-rate, strain hardening, geometry changes, etc.; and few studies have been done in this general area. An investigation of rectangular plates under uniformly distributed impulsive loading is the primary objective of this thesis and is undertaken to provide essential formation required for the development of approximate theories and design methods.

The relation of this work to Naval Engineering is clear: we will achieve improved understanding of the behavior of plates in response to grounding, slamming and underwater damages to the ship.

Since Rigid-Plastic Theory has not yet been applied to rectangular plates this study may be a tool in development of a new and simpler method in solving the rectangular plate problem.

Thesis Advisor : Norman Jones

Title : Assistant Professor of Naval Architecture

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NOMENCLATURE

σ_y = Yield stress (psi) = σ_o

E = Young's modulus

λ = $\dfrac{\rho\ V_o^2\ L_o^2}{\sigma_o\ H^2}$ = $\dfrac{\mu\ V_o^2\ L_o^2}{\sigma_o H^3}$

ρ = 0.098 lb/in^3 = density

V_o = Initial velocity (ft/sec)

L = Shorter side of the plate = 3.0 inches

H = Thickness of plate (inches

w_{max} = Maximum deflection of plate (inch)

μ = ρ H

w = Deflection of plate (inches)

δ_{max} = Maximum amplitude of pendulum swing

N = Number of cycles of pendulum swing

I_1 = Moment of inertia of the specimen = $m_1 D^2$

I_2 = Moment of inertia of the pendulum

D = Perpendicular distance from center of gravity of plate to the suspension point

$\bar{d}$ = Perpendicular distance from center of gravity of pendulum and plate

g = Acceleration of gravity

Θ_m = Maximum angular deflection of the pendulum

ω = Angular velocity of the pendulum

m_1 = Mass of specimen

m_2 = Mass of pendulum

INTRODUCTION

In this study the response of thin aluminum rectangular plates subjected to laterally applied dynamic loads is examined and experimental results are developed.

In some problems in which exact solutions are known, full allowance can be made for the elastic component of strain in the plastic region. However, more complex and realistic shapes of plates enforce the necessity for the development of approximate methods. Rigid-plastic theory has been used to analyze these complex problems. In the problem of four edges clamped rectangular plate, we are compelled with mathematical difficulties to disregard the elastic component of strain. For consistency, we must also disregard the purely elastic component of strain in the non-plastic region. In effect, therefore, we work with a material that is rigid when stressed below the yield point and in which Young's modulus has an infinitely great value. This hypothetical solid may be referred to as a plastic rigid material. In many technological forming processes (e.g. rolling, drawing, forging) experience shows that the assumption of a plastic-rigid material does not lead to any significant errors.[1]

Rigid-plastic theory has been used to analyze cantilevers,[2],[3] beams,[4],[5] circular,[6] annular,[7] and square[8] plates under

dynamic loads. Rigid-plastic analyses are simple because complex elastic-plastic behavior is ignored and replaced by perfectly plastic behavior. Experiments done on beams, cantilevers, circular, and annular plates indicate that Rigid-Plastic Theory is very accurate. It is possible to extend these theories to include the influence of finite deflections. Studies on annular plates indicate when the membrane effect is dominant the influence of axial restraints are much greater than material strain-rate sensitivity. Therefore, in such situations, strain-rate sensitivity effects can be disregarded which allows simpler solutions to be obtained.

For rectangular plates an exact Rigid-Plastic solution does not exist. The closest upper and lower bounds have been achieved by Wood[9] for static loads. Cox and Morland have solved the particular case of a simply-supported square plate loaded dynamically.

To the author's knowledge no previous studies have been undertaken on the behavior of rectangular plates subjected to dynamic loads sufficient to cause large permanent deformations. In view of this and knowing that the analytical work of rectangular plates would be very complex, it is clear, that approximate methods must be developed in order to analyze more complex and realistic shapes of plates.

The author remains hopeful that the results obtained from this study will be useful in development of a new method in solving the rectangular plate problem.

EXPERIMENTAL DETAILS

The tests reported in this thesis were carried out in the M.I.T. Aeroelastic and Structures Research Laboratory (Building 37-094). The ballistic pendulum shown in Figures 1 and 2 is attached to hinges on rails which in turn are clamped to the ceiling. Lead ballasts with different sizes were used in order to balance the pendulum and give the greatest allowable suring for each test.

In these tests Dupont Detasheet-D explosive and Dupont No.6 blasting capsules are used. Dimensions of the leader differed from test to test but the usual size was 1/8 inch by 12-20 inches.

A heat-sensitive-paper device was used in order to determine the amount of energy imparted to the pendulum. Heat-sensitive-paper is placed on the slightly curved surface of the device and a thin steel wire just touching the heat-sensitive-paper is attached to the end of pendulum. The amplitude of the swing determines the amount of energy imparted on the pendulum.

Specimens 8 inches by 6 inches were cut by band-saw and 8 holes were drilled in each specimen to provide the four edges clamped boundary condition (Figure 3). High strength steel bolts, nuts and washers were used to clamp the specimen on the head of

the pendulum. Grinding and polishing of the specimens were done manually making use of Silicon Carbide Papers (No. 240, 400, 600A and Crocus Cloth). A few drops of paraffin were applied on the specimen surface before the process to increase the cutting effect of silicon carbide paper.

In order to eliminate spalling, plastic waves, possible changes in material properties, high peak pressures in shock waves and spread of pressure waves (Appendix A) two methods were used. During the first few experiments a layer of 1/8 inch thick neoprene (weighing 42 grams with dimensions of 3 by 5 inches) was glued on the specimens underneath the explosive. Later this method was replaced with the fear that initial velocity might change due to the comparatively heavy neoprene. Foam rubber was cut (again having dimensions of 3 by 5 inches but weighing only 6 grams) and glued to the surface of each specimen underneath the explosive. Two layers of masking tape were attached to the side of the foam rubber facing the specimen in an attempt to obviate pitted surfaces on the specimen after explosion.

To increase the fixity of clamps serrations were machined on facing sides of the head (Figure 4).

To determine the actual (final) thicknesses of specimens after the polishing process fifteen readings, one inch apart from each other, from different locations on the plate were taken with

a micrometer. Percentage variation in thicknesses was found to be very small (0.25%). The average of these readings was accepted as the final thickness. Each specimen was weighed before each test and these weights are listed in Table 3.

The apparatus used in this project was prepared jointly by individuals* who did some dynamic experiments in the general program.

In this study Al 6061-T6 is used as specimen material. Chemical and Tensile tests (Tables 1 and 2) were done at M.I.T. to have full knowledge of the chemical and mechanical properties of the material.

The formula for initial velocity of the pendulum is derived from the conservation of energy and momentum considerations and initial velocity is determined using a computer program. (Appendix C and D).

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*The author; Sedat Tekin, LTJG, Turkish Navy; Roger Van Duzer, LT, USN; Robert Griffin, LT, USN.

EXPERIMENTAL DESCRIPTION AND DISCUSSION OF RESULTS

Experimental results (Table 3) show that deformed plates tend to have shapes similar to those of static collapse as shown in Figures 5 a-g and Table 4. The plates after deflection have a shape approximately of a pyramid.

For three different thicknesses of plates the relations between maximum deflections and initial velocity are plotted in Figure 6. (The formula for initial velocity is derived in Appendix C.) Differences in individual results may be due to unisotropic characteristics of the specimens. The unisotropy may arise from the different cutting locations of specimen on the Aluminum 6061-T6 sheet (Figure 7).

Nondimensionalized relation, w_{max}/H vs λ (Figure 8) shows that there is no evidence of shifting in the plot between different thicknesses. Therefore, it is evident that the strain-hardening has no effect in the character of the plot. Furthermore, aluminum is believed to be strain rate insensitive. Non-linear maximum deflection/H - λ relation is obtained for rectangular plates. When this result is compared to linear relation of square plates and the expected linear relation to rectangular plates the influence of the membrane effects in

rectangular plates becomes obvious.

Tests done with neoprene as energy absorber are closely related to those done with foam rubber. Since the difference between two results are consistent and negligibly small no considerable error is detected when either method is used.

No theoretical comparison with the experimental results is available since there is no existing pertinent theory. Cox and Morland only considered the case of a square plate and there does not appear to be a straightforward way to extend this analysis to examine rectangular plates.

CONCLUSIONS

An experimental study of the dynamic behavior of rectangular aluminum plates with four edges clamped and subjected to uniformly distributed impulses is presented in this thesis.

Tests have been done on rectangular Aluminum (6061-T6) plates with three different thicknesses. Deformations were measured (Table 4) and different relations were plotted on various graphs. From the results of tests done with neoprene and foam rubber as energy absorbers it is concluded that either material can be used for this purpose since neoprene bounces off after detonation just before the swing of pendulum starts, thus not affecting the initial velocity.

Results of this work should assist one in developing approximate analytical procedure to describe the behavior of rectangular plates.

APPENDIX A

A STUDY ON NEOPRENE AS AN ENERGY ABSORBER

Subjecting plates to impulsive loads may involve the action of spalling, plastic waves, the spread of pressure waves and high peak pressures in shock waves.[11] The effects of these actions are undesirable for sound investigations of finite deflections of plates. In order to prevent these influences some means of energy absorbtion is necessary. To be able to have a full understanding of the amount of energy dissipated by the material used and the effects of this dissipation on experimental results are essential. This study is an attempt to idealize a model to give better understanding of the importance of using an energy absorber in experiments done on plates subjected to dynamic loads.

In this study a NEOPRENE layer of 1/8 inch thick is idealized as a model. An experiment done without neoprene showed that the above mentioned various actions caused the destruction of the specimen whereas when the neoprene was used desirable deflections were obtained. From these experiments it is concluded that a considerable amount of incident pressure has been reflected by the neoprene while the rest was transmitted to the specimen.
To find a relation between the transmitted, reflected and incident energies let us consider the following mathematical analysis:

Initial velocity = u_o | ρ_o = density of air

Explosive cross section area = A | c_o = sound velocity in air

Mass of explosive = M | u = particle velocity

t = time

Assume explosion hits neoprene at t = 0,

for plane waves we have $p = \rho_o c_o u_o$ - pressure in surface layer of neoprene

at time "t" velocity of explosive waves is u(t) and pressure at neoprene surface is $p(t) = \rho_o c_o u$.

Differential equation, before the stress wave hits steel plate, is:[12]

$$M\frac{du}{dt} + p(t)\,A = 0$$

$$M\,\frac{du}{dt} = -\rho_o c_o uA$$

$$\frac{du}{u} = -\frac{\rho_o c_o A}{M}\,dt$$

$$\log u = -\frac{\rho_o c_o A}{M}t + B$$

Boundary condition $\rightarrow$ $u = u_o$ at $t = 0$ $\Rightarrow$ $B = \log u_o$

$$\therefore \log u = -\frac{\rho_o c_o A}{M}\,t + \log u_o$$

$$\log\left(\frac{u}{u_o}\right) = -\frac{\rho_o c_o A}{M}\,t$$

$$\frac{u}{u_o} = e^{-\frac{\rho_o c_o A}{M}t}$$

$$u = u_o \, e^{-\frac{\rho_o c_o A}{M} t}$$

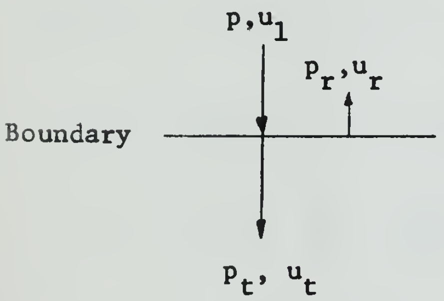
But $p = \rho_o c_o u$

$$\therefore p = \rho_o c_o u_o . e^{-\frac{\rho_o c_o A}{M} t} \quad \text{or} \quad p = p_o \, e^{-t/Q}$$

where, $p_o = \rho_o c_o u_o$ and $Q = \dfrac{M}{\rho_o c_o A}$

This equation represents the time history of pressure wave travelling along neoprene.

Now let us consider the clamped condition between neoprene and plate at which the pressure wave reaches the neoprene-aluminum boundary:



c = sound velocity (neoprene) = 177 ft/sec

ρ = density of neoprene

ρ_1 = density of aluminum

p_t = transmitted pressure

p_r = reflected pressure

u_t = part. velocity of trans. wave

u_r = part. velocity of reflect. wave

u_1 = part. velocity of initial wave

c_1 = sound velocity (Al) = 16700 ft/sec

for continuous displacement and velocity at the boundary, we have:

$$u_r = -(u_t - u_1) \qquad (1)$$

From force eqm at the boundary = $p + p_r = p_t$ (2)

We also have $p = \rho\ c\ u_1$ (3)

$$p_t = \rho_1\ c_1\ u_t \quad (4)$$

$$p_r = \rho\ c\ u_r \quad (5)$$

p and u_1 are known for the incident while u_t, p_t, u_r, p_r remain to be found.

From (1)

$$\left.\begin{aligned} u_r &= u_1 - u_t \\ u_r &= \frac{p_r}{\rho c} \end{aligned}\right\} \Rightarrow \quad u_1 - u_t = \frac{p_r}{\rho c}$$

$$u_t = u_1 - \frac{p_r}{\rho c} \quad (6)$$

$$\left.\begin{aligned} &\text{From (2)} \quad p_t = p + p_r \\ &\text{From (4)} \quad p_t = \rho_1\ c_1\ u_t \end{aligned}\right\} \quad p + p_r = \rho_1\ c_1\ u_t$$

$$u_t = \frac{p + p_r}{\rho_1 c_1} \quad (7)$$

Combine (6) and (7) $u_t = u_1 - \frac{p_r}{\rho c} = \frac{p + p_r}{\rho_1 c_1}$

$$u_1 - \frac{p}{\rho_1 c_1} = p_r \left(\frac{1}{\rho_1 c_1} + \frac{1}{\rho c}\right)$$

$$u_1 = \frac{p}{\rho c} \Rightarrow \frac{p}{\rho c} - \frac{p}{\rho_1 c_1} = p_r \left(\frac{1}{\rho_1 c_1} + \frac{1}{\rho c}\right)$$

$$p_r = p \left(\frac{\rho_1 c_1 - \rho c}{\rho_1 c_1 + \rho c}\right)$$

Let $\varepsilon = \frac{\rho c}{\rho_1 c_1}$ Then $p_r = p \left(\frac{1 - \varepsilon}{1 + \varepsilon}\right)$

Now $p_t = p + p_r$

$$p_t = p\left[1 + \left(\frac{\rho_1 c_1 - \rho c}{\rho_1 c_1 - \rho c}\right)\right] = p\left(\frac{\rho_1 c_1 + \rho c + \rho_1 c_1 - \rho c}{\rho_1 c_1 + \rho c}\right)$$

$$p_t = p\left(\frac{2\,\rho_1 c_1}{\rho_1 c_1 + \rho c}\right)$$

$$p_t = p\left(\frac{2}{1+\varepsilon}\right)$$

There is no change of time history of the waves at such reflections.

Since $\varepsilon = \dfrac{\rho c}{\rho_1 c_1}$ where $\rho = 0.098\ \text{lb/ft}^3$

$\rho_1 = 0.04\ \text{lb/ft}^3$

$\varepsilon = \dfrac{0.098 * 16700}{177 * 0.04} = 231$ $\quad c = 16700$ ft/sec

$c_1 = 177$ ft/sec

$$\therefore\ p_t = p\left(\frac{2}{232}\right) = .009\ p$$

which shows a great reduction in pressure wave. This simplified calculation of transmitted pressure wave proves that the influence of neoprene in impulsively loaded plate tests is of great importance. Therefore, we must use neoprene or some other substances (e.g. foam rubber) in order not to damage the testing specimens.

APPENDIX B

WEIGHT ESTIMATION

Pendulum = 44.814 lb.

Head	a.	Explosive face	5160 grams
	b.	Pendulum face	5125 grams

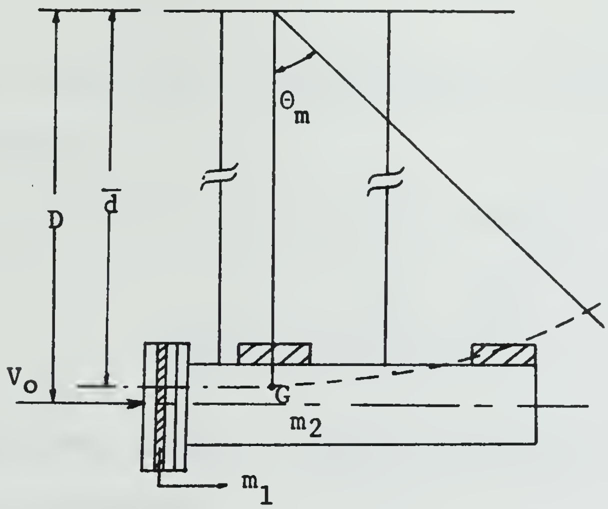
Lead Ballasts	#1	1815 grams
	#2	1780 grams
	#3	3205 grams
	#4	2099 grams

4 Steel Supports = 750 grams (see Figure 4)
(between head and pendulum)

Height of pendulum from ground to hinges = 11 ft. 6 in.

APPENDIX C

DETERMINATION OF INITIAL VELOCITY OF THE PENDULUM[10]



A combination of direct and rotational impulse occurs in the use of the ballistic pendulum. When the detasheet explosive is detonated it initiates rotation of the pendulum. The observed amplitude of this motion can then be used to calculate the initial velocity V_o. If D is the perpendicular distance from center of gravity of plate to the suspension point, Θ_m the maximum angular deflection of the pendulum and ω is it's angular velocity immediately after impulse, and if m_1, m_2, and $I_1 = m_1D^2$, I_2 represent the masses and moments of inertia about the pivot of the specimen and pendulum respectively then,

$$I_2\omega = Dm_1(V_o - D\omega) \qquad (1\text{-a})$$

$$V_o = \frac{I_2 + m_1D^2}{m_1D}\,\omega \qquad (1\text{-b})$$

The law of conservation of energy applied to this process yields the relation

$$\left.\begin{aligned} \frac{1}{2}\,[\,I_2 + m_1D^2\,]\,\omega^2 &= g\,(m_1 + m_2)\,\bar{d}\,(1 - \mathrm{Cos}\,\Theta_m) \\ &= 2g\,(m_1 + m_2)\,\bar{d}\;\mathrm{Sin}\,\frac{\Theta_m}{2} \end{aligned}\right\} \quad (2)$$

where $\bar{d}$ is the distance from the center of gravity of pendulum and specimen to the support.

Substitution of Equation (2) in Equation (1-b) determines the initial velocity V_o as

$$V_o = \frac{\sqrt{2g(I_2 + m_1\,D^2)\,\bar{d}\,(\,1 - \mathrm{Cos}\,\Theta_m\,)(m_1 + m_2)}}{m_1D}$$

or $\quad V_o \simeq \left(\frac{2}{m_1}\right)\left(\mathrm{Sin}\,\frac{\Theta_m}{2}\right)\sqrt{(gI_2m_2)/D}$

when $\quad \bar{d} \simeq D \quad$ and $m_1 << m_2$

A computer program was written using Equation (3) to evaluate the initial velocity for each test and the results are presented in Appendix D.

APPENDIX D

COMPUTER PROGRAM TO CALCULATE V_o

```
FORTRAN IV G LEVEL 1, MOD 3     MAIN     DATE= 19/10/06       PAGE 0001

100     FORMAT(7F10.2)
300     FORMAT(5X,'E=',F10.5,2X,'A=',F10.5,2X,'W=',F10.5,2X,'V=',F20.5)
10      READ(5,100)BER,B,ASS,TL,Y,BH,TH
        BTA=TL/133.425
        Z=ASS+B+29431.887
        CG=((Z-B)*2.4875+B*5.975)/Z
        X=TH*BH*44.814*5.0625
        D=138.8-Y-CG
        BL=136.3125-Y
        ERT=X*BL**2.
        ERTB=Z*D**2.
        AA=SQRT(772.08*(ERTB+ERT)*D*(1.-COS(BTA))*(X+Z))
        V=AA/(X*BL*12.0)
        WRITE(6,300)VER,BTA,X,V
        IF(BER.NE.0.0)GO TO 10
        CONTINUE
        RETURN
        END
```

```
E= III 14    A=  0.03232    W=  127.81900    V=  183.01584
E= III 1     A=  0.04591    W=  128.02316    V=  259.71631
E= II  14    A=  0.03841    W=   83.71524    V=  330.42529
E= II  1     A=  0.03092    W=   83.10275    V=  267.95264
E= III 15    A=  0.06980    W=  127.95511    V=  395.32813
E= III 16    A=  0.07401    W=  128.22734    V=  417.87256
E= III 12    A=  0.04075    W=  127.68282    V=  226.56285
E= III 17    A=  0.03466    W=  127.88698    V=  176.96474
E= III 24    A=  0.07307    W=  128.33626    V=  412.40063
E= III 22    A=  0.03584    W=  128.65613    V=  201.57072
E= IV  1     A=  0.04731    W=  166.35529    V=  190.44037
E= IV  2     A=  0.03701    W=  166.38248    V=  148.92992
E= IV  8     A=  0.07214    W=  166.38248    V=  331.59277
E= IV  4     A=  0.07542    W=  166.20551    V=  346.88477
E= IV  5     A=  0.06464    W=  166.41655    V=  297.13306
E= IV  7     A=  0.06839    W=  166.38248    V=  275.29224
E= II  12    A=  0.02928    W=   83.10275    V=  251.86386
E= II  17    A=  0.04591    W=   83.91949    V=  389.20361
E= II  15    A=  0.02717    W=   83.57913    V=  230.99631
```

E=	III	13	A=	0.03185	W=	128.15927	V=	180.02188
E=	I	14	A=	0.03118	W=	60.77867	V=	368.09546
E=	I	1	A=	0.03876	W=	60.43835	V=	459.72339
E=	II	13	A=	0.04684	W=	83.23886	V=	405.02759
E=	I	2	A=	0.03185	W=	60.84676	V=	375.71802

Where, E = Specimen number

A = Angle pendulum swing (radians)

W = Weight of target plate (grams)

V = V_o (ft/sec)

and

I - .089 in plate

II - .1225 in plate

III - .189 in plate

IV - .244 in plate

TABLE 1

REPORT OF CHEMICAL ANALYSIS

Description of Samples: Al 6061 - T6

SAMPLE	% Si	% Cu	% Mg	% Cr
11/.1225	0.65	0.20	0.85	0.26
11/.189	0.61	0.21	0.83	0.21
13/.089	0.57	0.24	0.89	0.23
.25	0.60	0.24	1.04*	0.17

* Examination of the chemical-test results shows a percentage increase of magnesium in the composition of 0.244 inch thick plates. However, Al 6061-T6 may contain a maximum of 1.20% magnesium without the mechanical properties of the alloy being changed[13]. The chemical test for the plates of different thicknesses are found to lie under this limit resulting with the fact that the plates used in the experiments have the same mechanical behavior (i.e. same elongation, hardness, fatigue endurance limit and work hardening in annealed or solution-heat-treated condition.[14]) In conclusion, it is certain that higher percent Mg has no effect or influence on the 0.244 inch plate.

TABLE 2

TENSILE TEST RESULTS FOR 6061 - T6 ALUMINUM

All Samples 1/2 inches by 9 inches by thickness

SAMPLE	THICKNESS	σ yield(psi)	E(psi)
16	0.089	41800*	9.85 (10^6)
15	.089	37800*	10.8 (10^6)
7	.1225	41300	10.5 (10^6)
9	.1225	40500	10.85 (10^6)
8	.1225	41700	11.68 (10^6)
4	.187	40100	10.75 (10^6)
6	.189	41400	9.78 (10^6)
19	.246	41500	10.8 (10^6)
18	.246	41400	9.4 (10^6)

* Differences in individual results may be due to unisotropy of the specimens since they are cut from different parts of the Aluminum sheets as shown in Figure 7.

The yield stress is the stress at which the material exhibits either a specified limiting deviation from the proportionality of stress to strain. When the deviation is expressed in terms of an increase in strain above the proportional value the result is termed an offset yield strength. The offset is usually specified to be 0.2% (a plastic strain of .002) for steel and aluminum alloys.[15] Therefore, in order to determine yield strength of the material used in experiments an offset of .2% is used.

TABLE 3

BASIC INFORMATION OF EXPERIMENTAL RESULTS

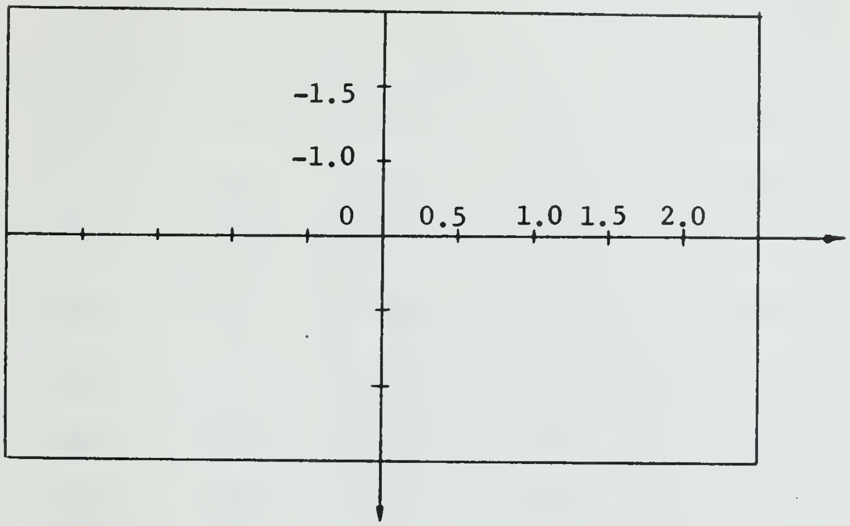
Specimen No.	Weight (gr)	H(in)	δ(in)	w_{max}(in)	w_{max}/H(in)	V_o(ft/sec)	λ	Neoprene or Foam	Comments
1	575	.24442	6 5/16	.109	.446	190.44037	12.8	F	O.K.
2	573	.24446	4 15/64	.047	.192	148.92992	7.9	F	O.K.
4	573	.2442	10 1/16	.325	1.33	346.88477	42.7	F	O.K.
5	575	.24451	8 5/8	.225	.92	297.13306	31.7	F	O.K.
7	576	.24446	9 1/8	.214	.87	275.29224	27.1	F	O.K.
8	580	.24446	9 20/32	.297	1.211	331.59277	39.2	F	O.K.
1	436	.1881	6 4/32	.227	1.205	259.71631	41.0	N	Good results
12	438	.1876	5 7/16	.192	1.025	226.56285	30.6	F	O.K.
13	435	.1883	6 8/32	-	-	180.02188	-	without N or F	Spalling and shear
14	437	.1878	4 10/32	.158	.843	183.01584	20.5	N	Good results
15	437	.1880	9 5/16	.3765	2.000	395.32813	95.25	F	O.K.
16	440	.1884	9 7/8	.454	2.41	417.87256	106.5	F	Shear
17	440	.1879	4 5/8	.151	.805	176.96474	19.1	F	O.K.
22	444	.18903	4 25/32	.158	.837	201.57072	24.7	F	O.K.
24	440	.18856	9 3/4	.418	2.22	412.40063	103.8	F	O.K.
1	278	.1221	4 8/64	.289	2.362	267.95264	103.0	N	O.K.

Table 3 - continued

Specimen No.	Weight (gr)	H(in)	δ(in)	w_{max}(in)	w_{max}/H(in)	V_o(ft/sec)	λ	Neoprene or Foam	Comments
12	281	.1221	3 29/32	.271	2.205	251.86386	90.6	F	O.K.
13	282	.1223	6 1/4	-	-	405.02759	-	F	Slipped
14	281	.1230	5 8/64	.350	2.842	330.42529	157.0	N	Slightly slipped
15	287	.1228	3 5/8	.222	1.82	230.99631	76.2	F	O.K.
16	281	.1226	10 12/64	-	-	-		N	Excessive charge ripped off the plate
17	288	.1233	6 1/8	.429	3.48	389.20361	216	F	O.K.
1	182	.0888	5 11/64	-	-	459.72339	-	N	Slipped
2	180	.0894	4 1/4	-	-	375.71802		F	Very slight slip
14	185	.0893	4 10/64	-	-	368.09546		N	Slipped

TABLE 4

FINAL DEFORMATION OF PLATES



Specimen No. 15 Thickness = .188 inches

y / x	-1.0	-0.5	0	0.5	1.0
-2.0	0.077	0.134	.145	.130	.078
-1.5	.117	.223	.260	.225	.118
-1.0	.140	.264	.3285	.262	.141
-0.5	.158	.295	.361	.292	.160
0	.179	.304	.3765*	.305	.175
0.5	.161	.295	.360	.2935	.161
1.0	.140	.259	.322	.260	.141
1.5	.1195	.221	.258	.219	.1175
2.0	.078	.131	.147	.132	.079

* .3765 = maximum deflection of plate, w_{max} (inches)

Table 4 - continued

Specimen No. 17 Thickness =.1879 inches

x \ y	-1.0	-0.5	0	0.5	1.0
-2.0	.0195	.0355	.045	.036	.0195
-1.5	.040	.076	.091	.075	.040
-1.0	.05	.097	.123	.097	.052
-0.5	.058	.111	.141	.113	.060
0	.0625	.120	.151*	.120	.063
0.5	.057	.112	.143	.114	.060
1.0	.050	.098	.122	.095	.051
1.5	.039	.077	.092	.075	.040
2.0	.0215	.039	.046	.040	.023

Specimen No. 22 Thickness = .189 inches

x \ y	-1.0	-0.5	0	0.5	1.0
-2.0	.020	.0415	.047	.045	.022
-1.5	.042	.078	.095	.080	.049
-1.0	.056	.103	.127	.110	.064
-0.5	.067	.121	.147	.127	.070
0	.069	.130	.158*	.133	.074
0.5	.067	.123	.150	.1235	.069
1.0	.058	.105	.131	.108	.063
1.5	.042	.080	.100	.085	.047
2.0	.021	.037	.05	.047	.026

*Maximum deflection

Table 4 - continued

Specimen No. 1 Thickness = .2444 inches

x \ y	-1.0	-0.5	0	0.5	1.0
-2.0	.013	.024	.031	.026	.012
-1.5	.026	.0495	.065	.050	.025
-1.0	.035	.067	.086	.068	.036
-0.5	.040	.079	.102	.080	.044
0	.043	.087	.109*	.0875	.046
0.5	.040	.080	.104	.082	.045
1.0	.033	.070	.090	.069	.036
1.5	.024	.052	.069	.056	.026
2.0	.0125	.027	.036	.029	.015

Specimen No. 5 Thickness =.24451 inches

x \ y	-1.0	-0.5	0	0.5	1.0
-2.0	.032	.056	.0645	.0565	.032
-1.5	.058	.1185	.132	.1135	.058
-1.0	.079	.149	.179	.143	.076
-0.5	.090	.171	.2145	.168	.087
0	.095	.177	.225*	.173	.093
0.5	.090	.170	.208	.162	.088
1.0	.074	.147	.177	.139	.073
1.5	.0575	.115	.126	.109	.058
2.0	.0315	.058	.066	.0585	.0315

*Maximum deflection

Table 4 - continued

Specimen No. 8 Thickness = .24446 inches

x \ y	-1.0	-0.5	0	0.5	1.0
-2.0	.049	.0765	.093	.078	.0482
-1.5	.084	.160	.187	.1602	.090
-1.0	.1085	.200	.2465	.203	.110
-0.5	.116	.2245	.285	.234	.1308
0	.121	.233	.297*	.2424	.132
0.5	.114	.225	.282	.231	.1305
1.0	.105	.194	.241	.203	.111
1.5	.0835	.151	.183	.160	.089
2.0	.050	.0785	.096	.081	.051

Specimen No. 7 Thickness = .24446 inches

x \ y	-1.0	-0.5	0	0.5	1.0
-2.0	.0385	.067	.074	.065	.367
-1.5	.072	.123	.140	.1205	.070
-1.0	.091	.160	.189	.158	.089
-0.5	.105	.184	.202	.182	.100
0	.108	.194	.214*	.194	.1075
0.5	.105	.185	.204	.182	.101
1.0	.0905	.159	.190	.158	.091
1.5	.0715	.1265	.1425	.121	.0691
2.0	.038	.065	.073	.0648	.039

*Maximum deflection

Table 4 - continued

Specimen No. 14 Thickness = .123 inches

x \ y	-1.0	-0.5	0	0.5	1.0
-2.0			.157		
-1.5			.258		
-1.0			.317		
-0.5			.336		
0	.174		.350*	.297	.177
0.5			.341		
1.0			.320		
1.5			.2645		
2.0			.259		

Specimen No. 1 Thickness = .1221 inches

x \ y	-1.0	-0.5	0	0.5	1.0
-2.0			.140		
-1.5			.230		
-1.0			.273		
-0.5			.284		
0	.145	.253	.289*	.253	.145
0.5			.285		
1.0			.275		
1.5			.232		
2.0			.142		

*Maximum deflection

Table 4 - continued

Specimen No. 14 Thickness = .1878 inches

y / x	-1.0	-0.5	0	0.5	1.0
-2.0			.0585		
-1.5			.107		
-1.0			.1355		
-0.5			.149		
0	.082	.133	.158*	.133	.079
0.5			.151		
1.0			.136		
1.5			.106		
2.0			.565		

Specimen No. 1 Thickness =.1881 inches

y / x	-1.0	-0.5	0	0.5	1.0
-2.0			.067		
-1.5			.141		
-1.0			.172		
-0.5			.212		
0	.117	.2185	.227*	.2165	.118
0.5			.215		
1.0			.211		
1.5			.143		
2.0			.072		

*Maximum deflection

Table 4 - continued

Specimen No. 16 Thickness = .1884 inches

y / x	-1.0	-0.5	0	0.5	1.0
-2.0			.185		
-1.5			.323		
-1.0			.395		
-0.5			.436		
0	.222	.381	.454*	.376	.220
0.5			.439		
1.0			.396		
1.5			.323		
2.0			.186		

*Maximum deflection

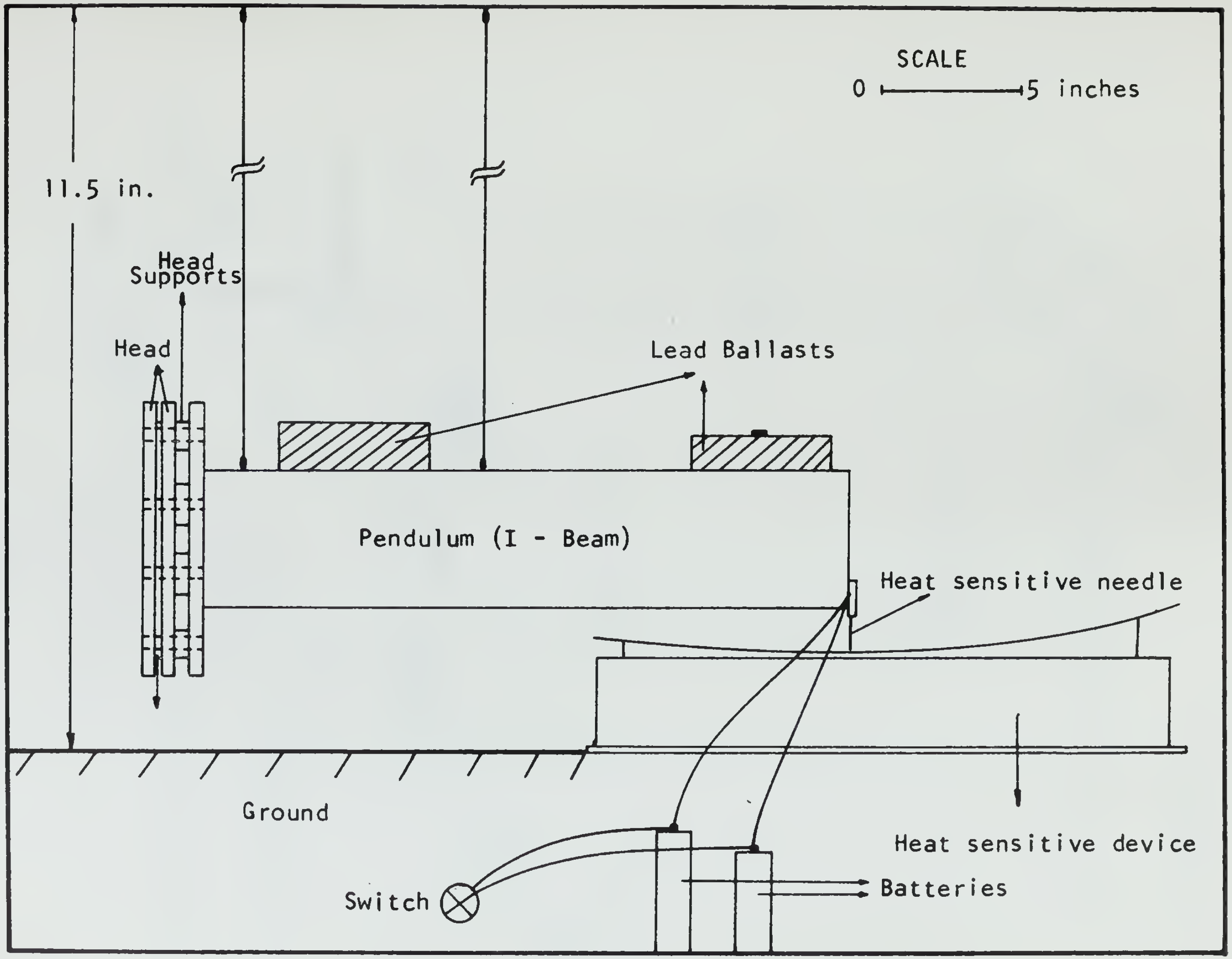


FIGURE I

THE BALLISTIC PENDULUM

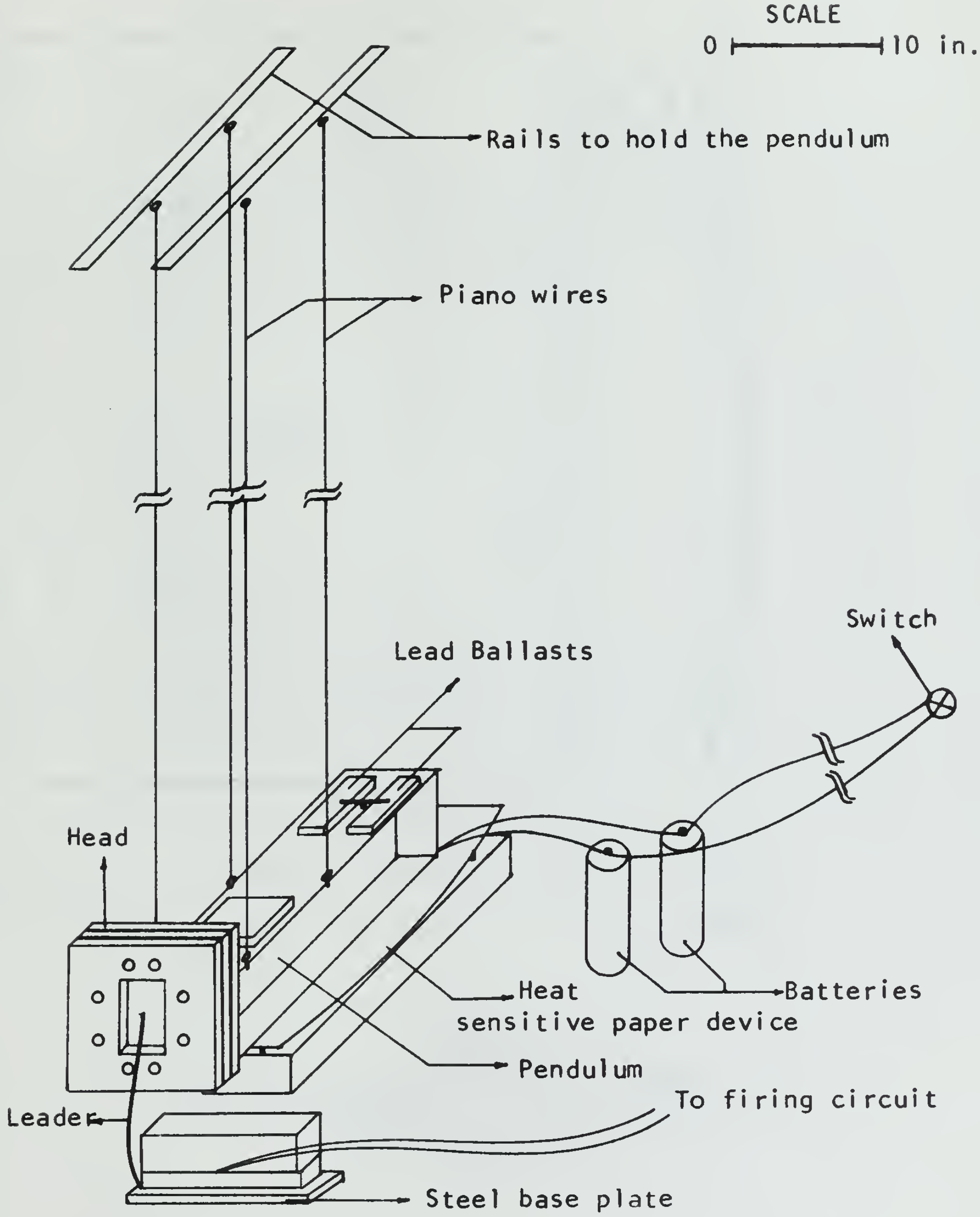


FIGURE 2

EXPERIMENTAL APPARATUS

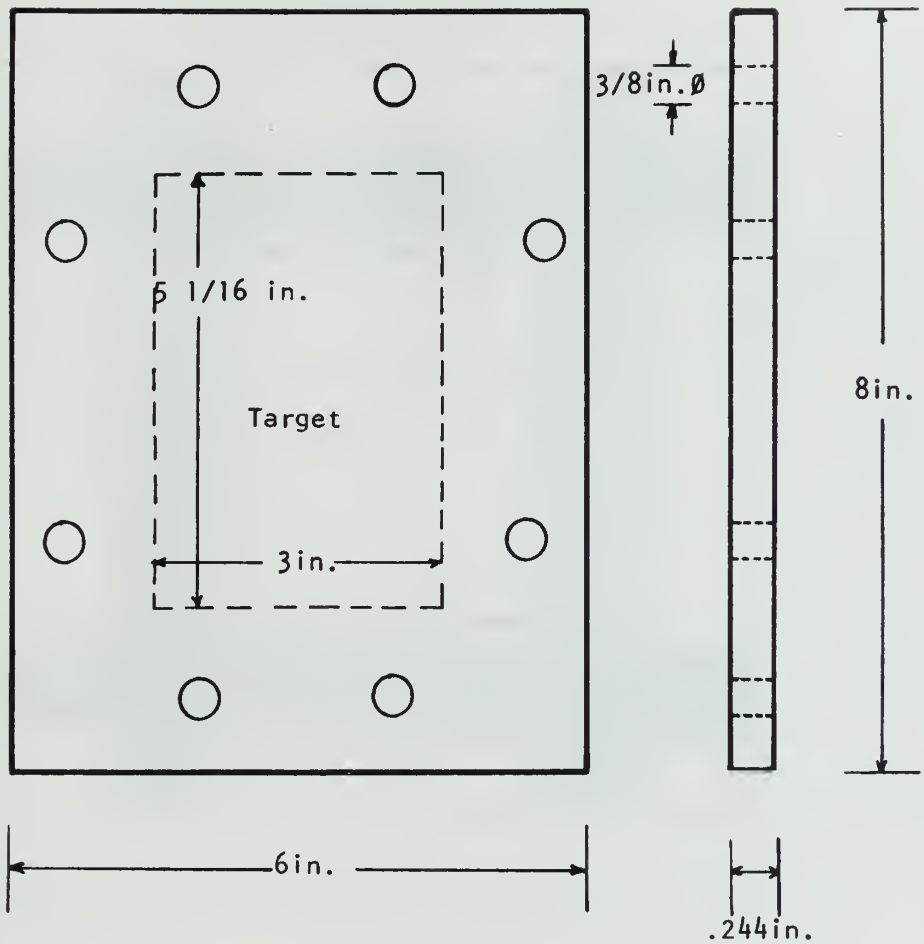


FIGURE 3

PHYSICAL CHARACTERISTICS OF AL. 6061 - T6

RECTANGULAR PLATE

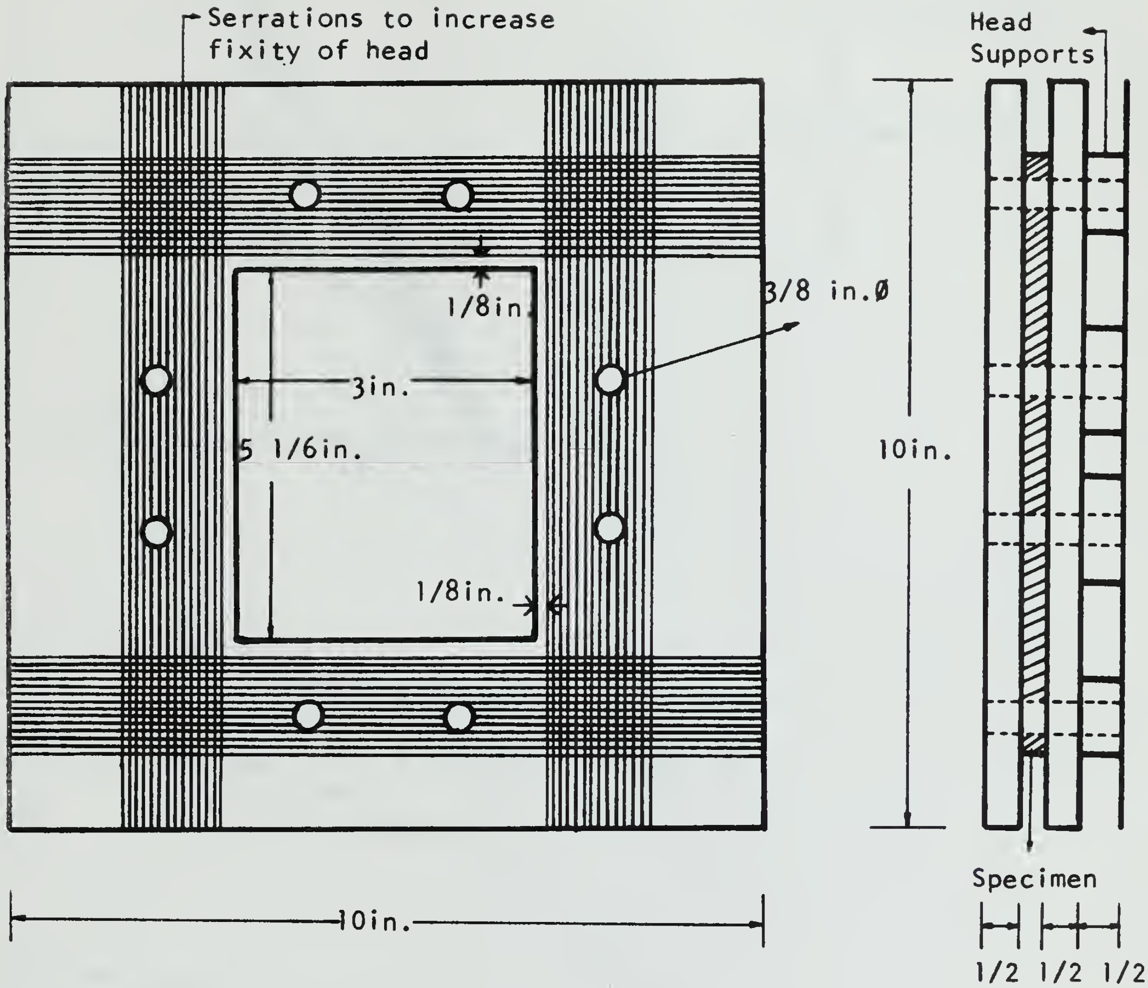


FIGURE 4

HEAD OF BALLISTIC PENDULUM

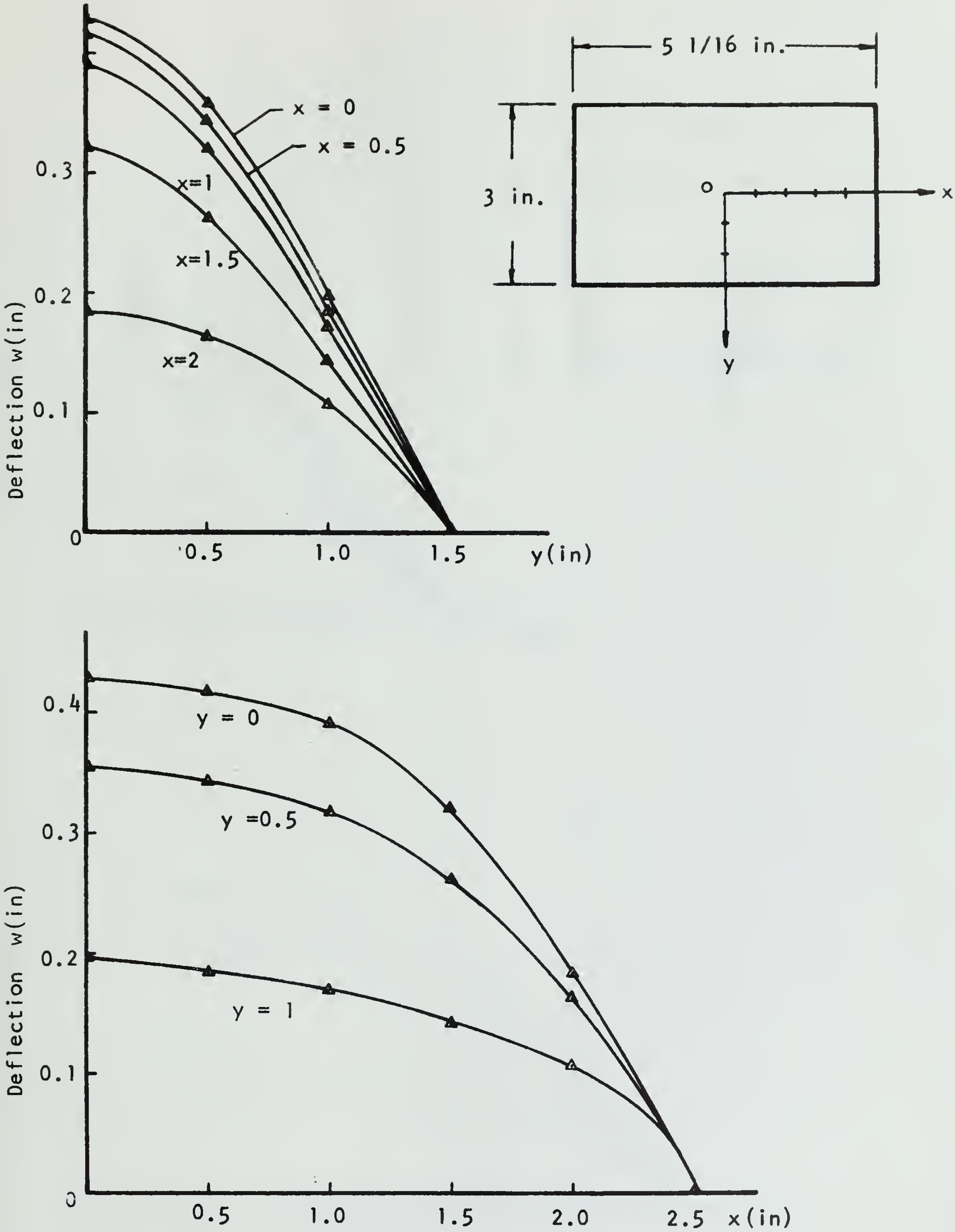


FIGURE 5-A

DEFLECTION OF ALUMINUM SPECIMEN NO. 17

H = .1223

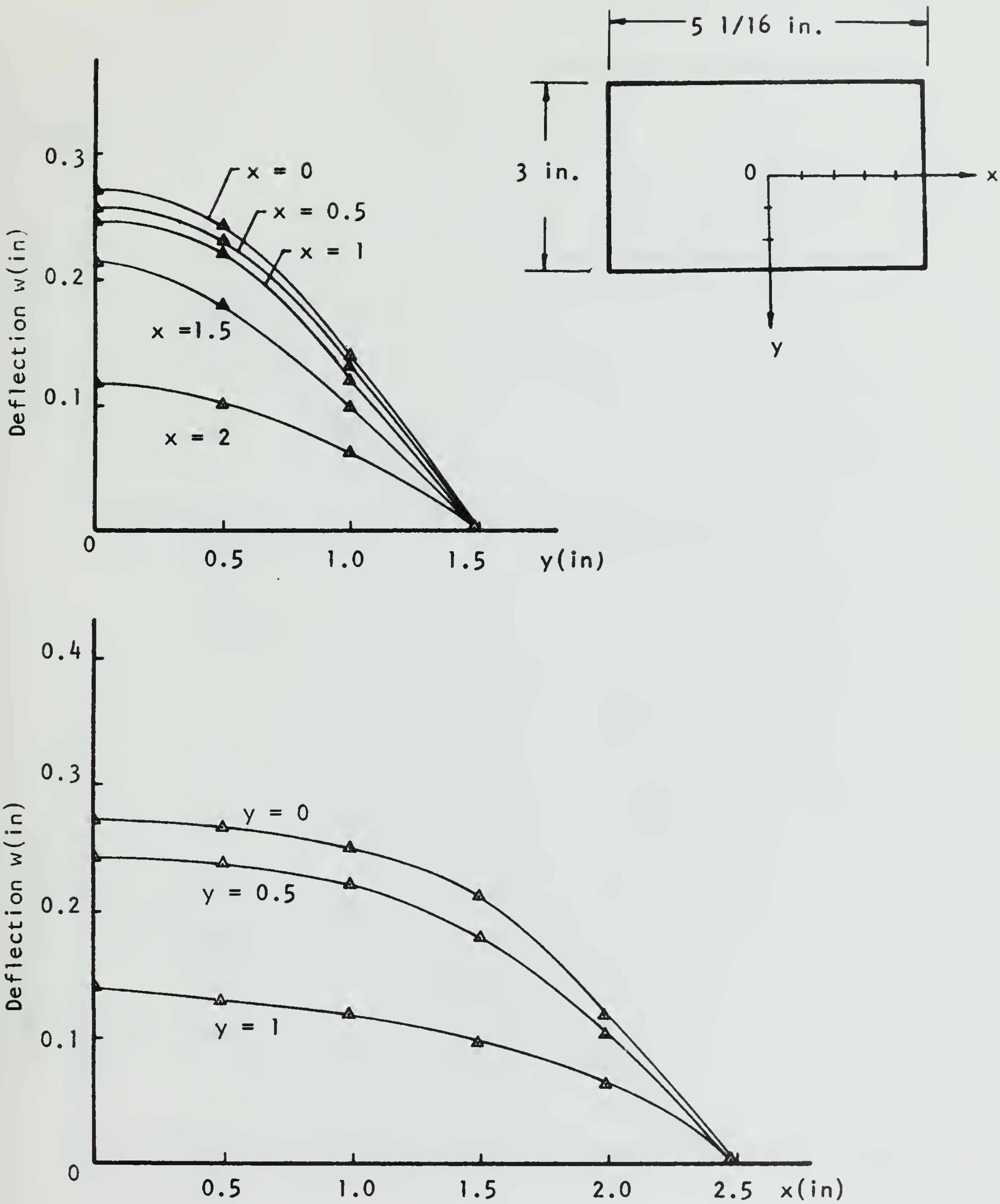


FIGURE 5-B

DEFLECTION OF ALUMINUM SPECIMEN NO. 12

H - .1221

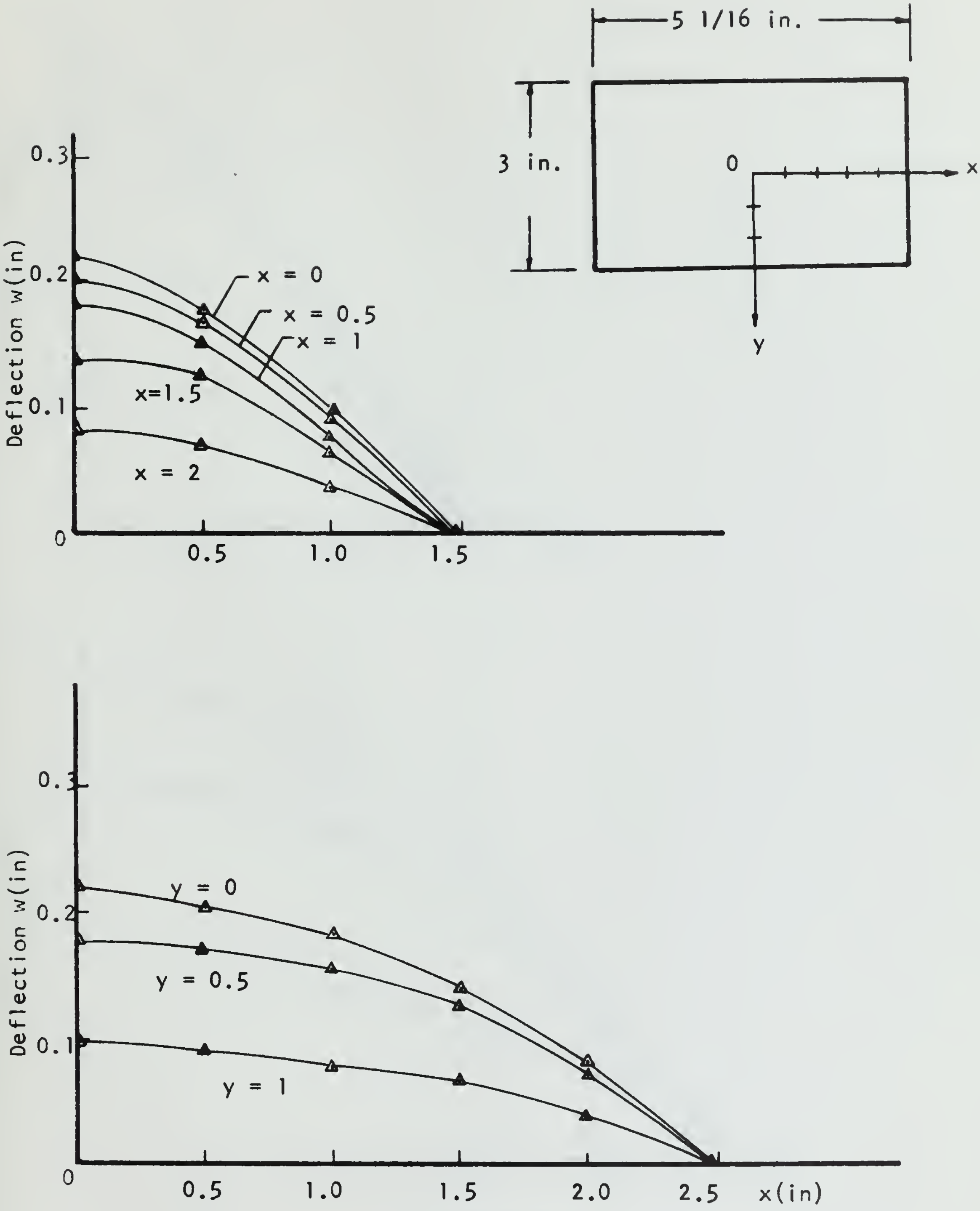
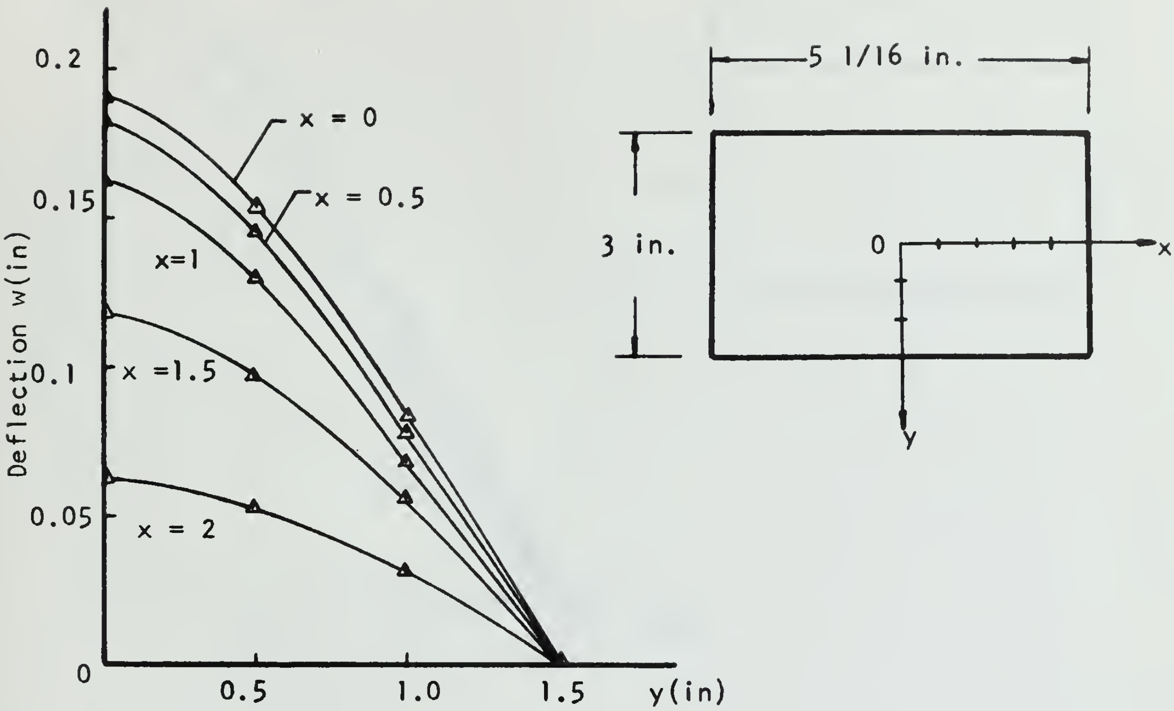


FIGURE 5-c

DEFLECTION OF ALUMINUM SPECIMEN NO. 15

H = .1228



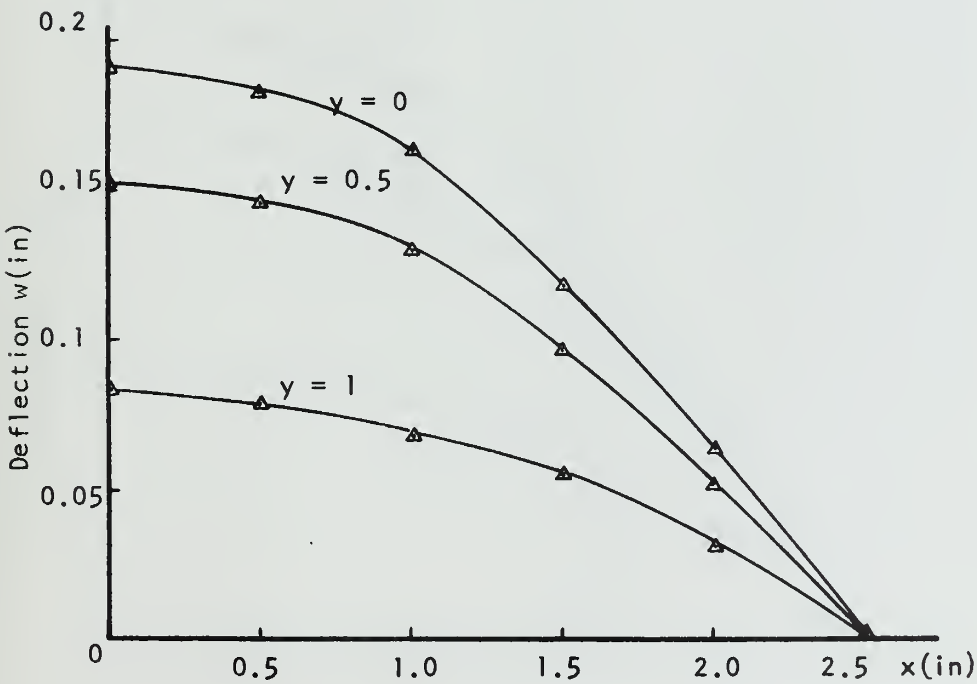


FIGURE 5-D

DEFLECTION OF ALUMINUM SPECIMEN NO. 12

H = .1876

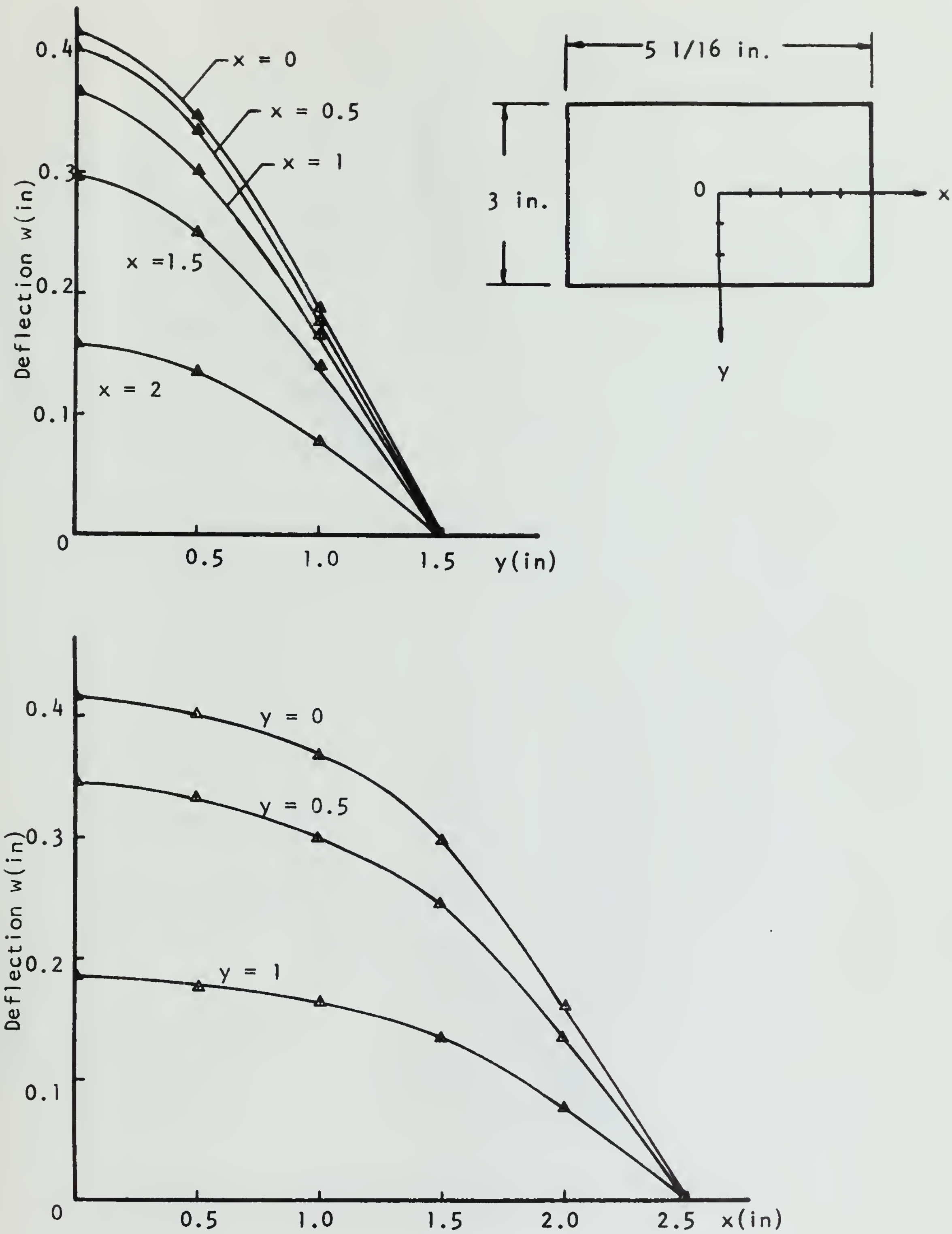


FIGURE 5-E

DEFLECTION OF ALUMINUM SPECIMEN NO. 24

H = .18856

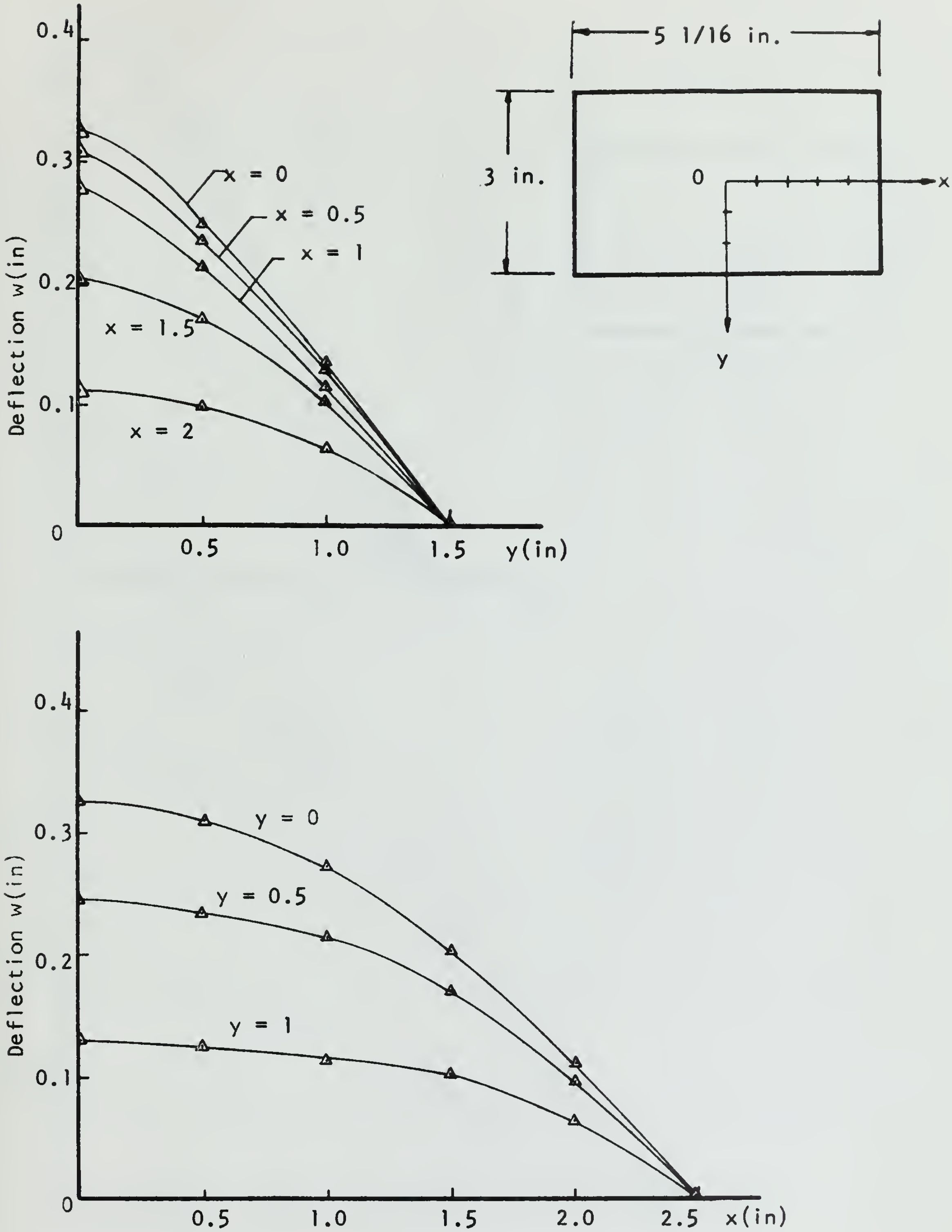


FIGURE 5-F

DEFLECTION OF ALUMINUM SPECIMEN NO. 24

H = .2442

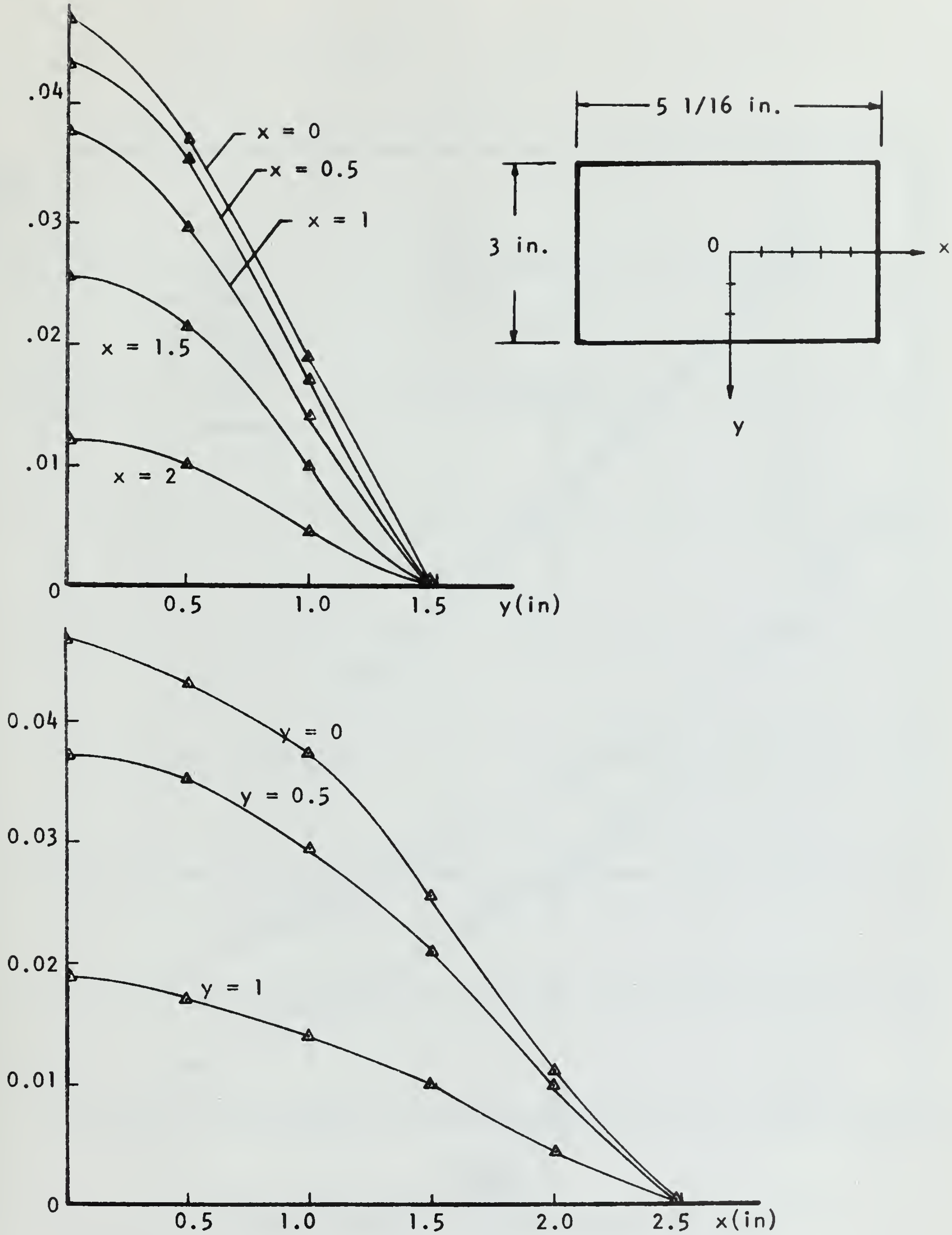


FIGURE 5-G

DEFLECTION OF ALUMINUM SPECIMEN NO. 2

H = .24446

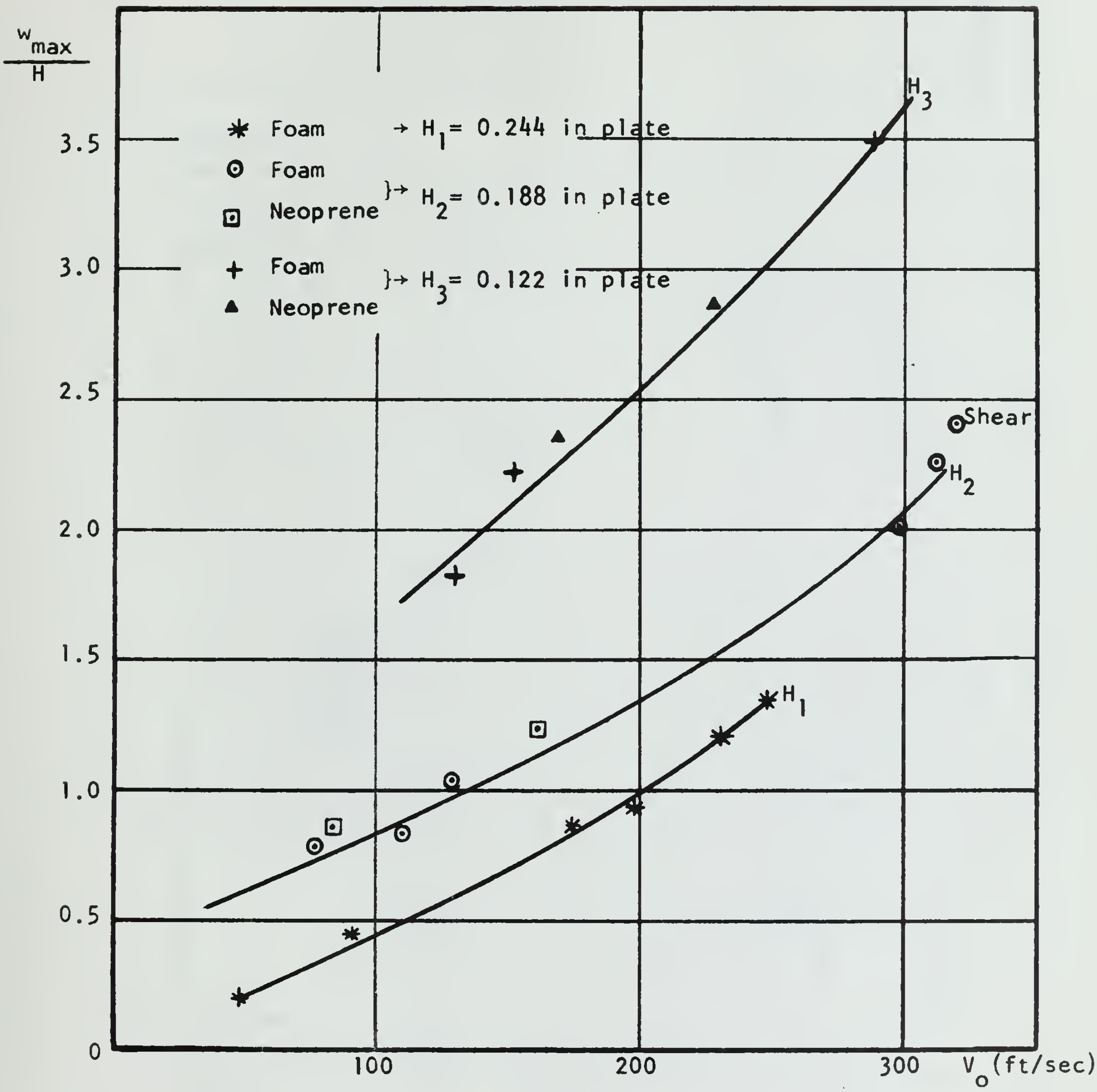


FIGURE 6

MAXIMUM DEFLECTION - INITIAL VELOCITY RELATIONS

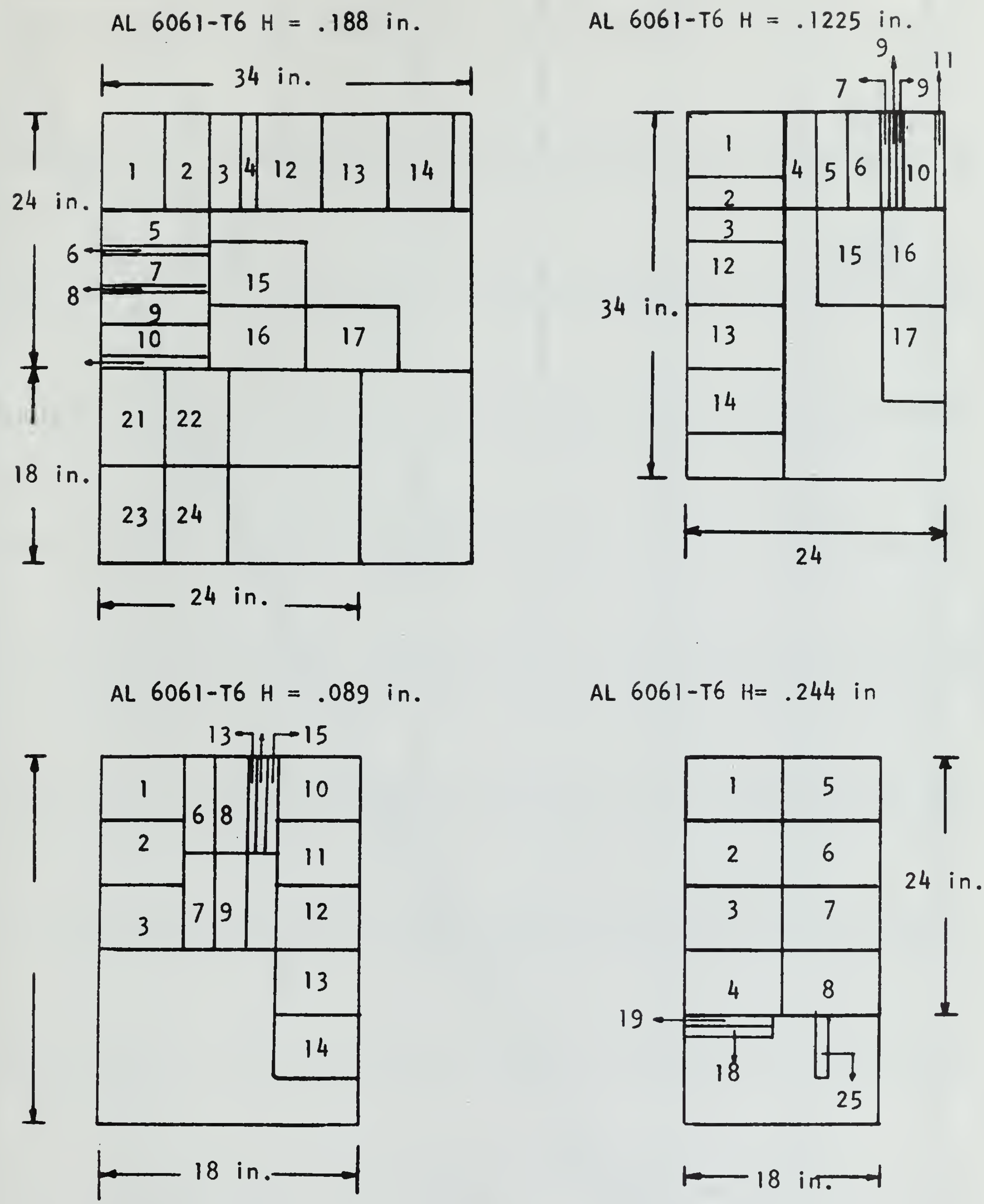


FIGURE 7

LOCATIONS OF SPECIMENS ON ALUMINUM SHEETS

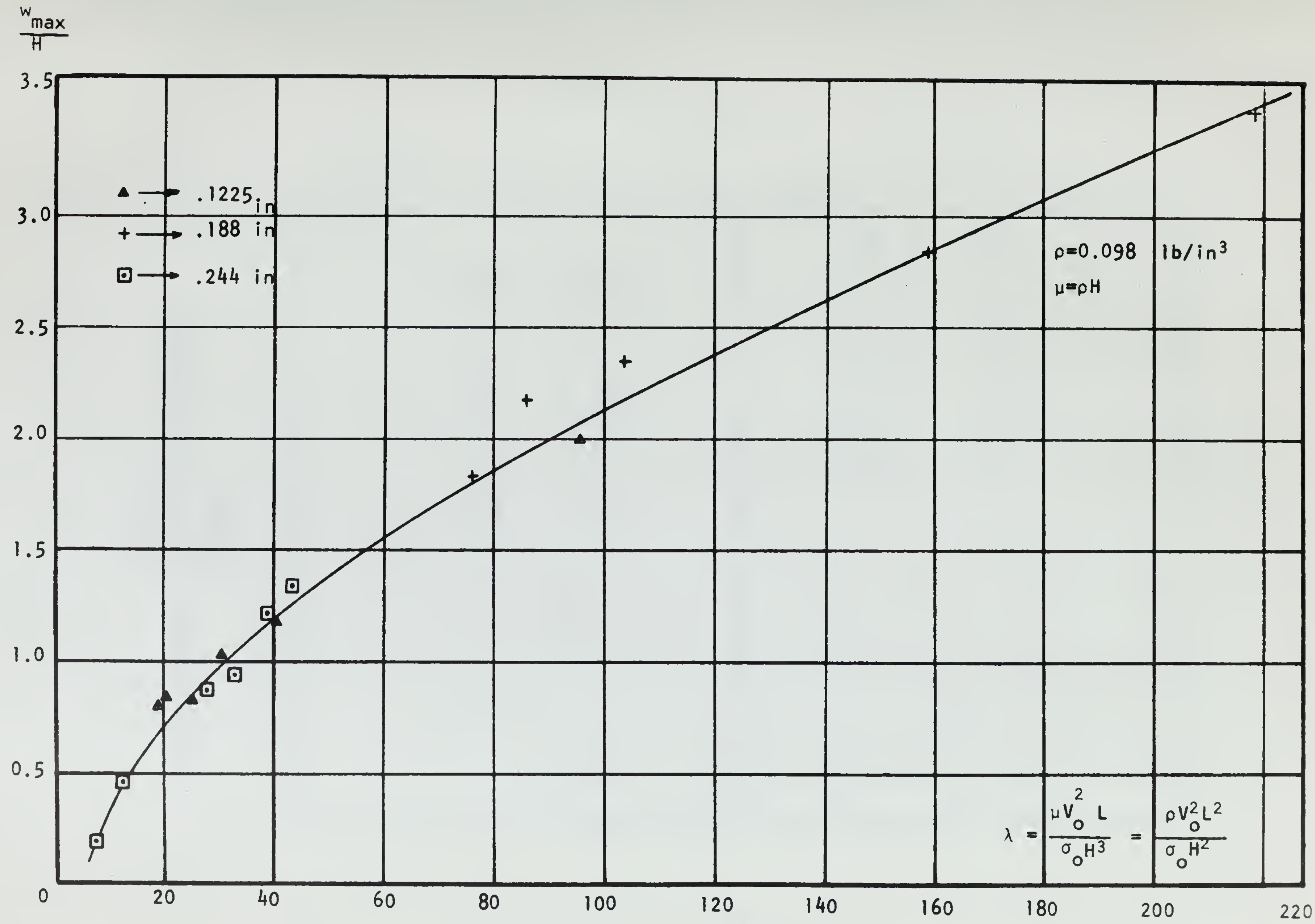


FIGURE 8

NONDIMENSIONALIZED RELATION OF w_{max}/H VS λ

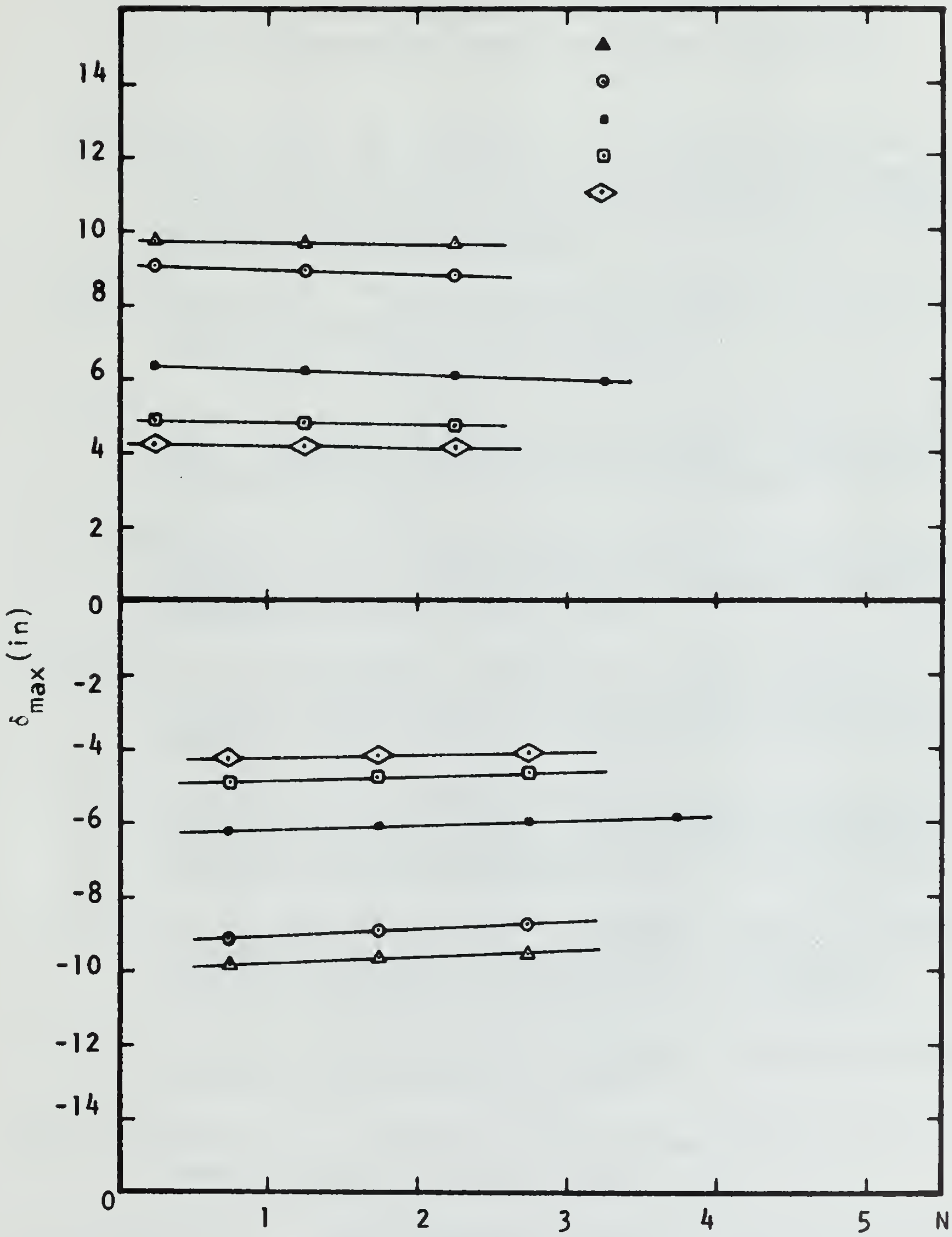


FIGURE 9

PENDULUM MAXIMUM SWING - NUMBER OF CYCLES RELATIONS

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